

MEASURING COMPLEX DIELECTRIC CONSTANT OF DIELECTRIC SLAB-COATED CONDUCTORS IN MILLIMETER WAVES BY RADIATING TECHNIQUE

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ABSTRACT

This paper describes a novel method for determining complex dielectric constant of dielectric slab-coated conductors. The constant may be found by measuring an angle  $\theta_m$  of maximum radiation and two radiation angles  $\theta_{1h}$ ,  $\theta_{2h}$  of half peak power from the leaky wave antenna. The method is specially useful for millimeter waves and can realize automatical measurement.

INTRODUCTION

In the past the determination of complex dielectric constant of microstrip substrate has often been accomplished via experimental means, mostly by the resonance schemes. However these methods can be used only in microstrip with thin substrate. In this paper we describe novel method for determining complex dielectric constant of dielectric slab-coated conductors. The new method is referred to as a radiating technique. The technique is specially useful for millimeter waves. Essential principle of the technique is as follows.

A leaky wave structure composed of an array of thin metallic rectangular strip is made of a dielectric slab-coated conductors. Complex dielectric constant  $\epsilon_r$  of the slab is still unknown. The geometry of the leaky wave structure is shown in Fig. 1. The thickness and the length of the dielectric slab are  $h$  and  $L$ , respectively,  $d$  is the length of the

period,  $W$  is width of the strip. A horn is used as feed of the structure. Complex dielectric constant of the dielectric slab may be found by measuring an angle  $\theta_m$  of maximum radiation and two radiation angles  $\theta_{1h}$  and  $\theta_{2h}$  of half peak power from the leaky wave antenna.

ANALYSIS

The structure shown in Fig. 1 is a periodically loaded, basically slow, traveling wave one. First, we find a relation between the parameters  $h$ ,  $d$ ,  $W$ ,  $\epsilon_r$  and the complex propagation constant  $k_z$  along the leaky wave structure where  $k_z = \beta + j\alpha$ ,  $\beta$  and  $\alpha$  represent corresponding phase and attenuation constant. We employ Green's function method in the Fourier transform of spectral domain. Referring to Fig. 1, let us consider a  $y$ -polarized wave traveling along the  $z$ -direction. The dielectric slab is supposed to be infinite in the  $y$ - $z$  plane. In this case the expression for  $\tilde{G}$ , the Fourier transform of spatial-domain Green's function for the  $E$ -field of  $y$ -direction line source located at a distance  $x_0$  above the dielectric surface (referring to Fig. 2), is given by (1)

$$\tilde{G}(x, \zeta) = \frac{\mu_0}{j2k_{x1}} (\exp(-jk_{x1}|x-x_0|) + R \exp(-jk_{x1}|x+x_0|)) \quad (1a)$$

where

$$R = \frac{jk_{x_1} - k_{x_2} \operatorname{ctg}(k_{x_2} h)}{jk_{x_1} + k_{x_2} \operatorname{ctg}(k_{x_2} h)}$$

$$k_{x_1}^2 = k_0^2 - \xi^2, \quad k_{x_2}^2 = \epsilon_r k_0^2 - \xi^2$$

$k_0$ : free-space propagation constant.

In our case the line source is located at the dielectric surface,  $x_0 = 0$ . So expression (1a) has following simple form

$$\tilde{G}(x, \xi) = \frac{\mu_0 \exp(-jk_{x_1} x)}{jk_{x_1} + k_{x_2} \operatorname{ctg}(k_{x_2} h)} \quad (1b)$$

The current distribution on the strip in the  $z$ -direction is supposed to be (2)

$$J_y(z) = \frac{\exp(-jk_z z)}{k_0 \sqrt{w^2 - \bar{z}^2}}$$

$$+ \sum_{v=0}^{+\infty} (-j)^v A_v \cos(W_v(\bar{z} + w)) \quad (2)$$

where  $\bar{z} = z - nw$  ( $n = 0, \pm 1, \pm 2, \pm 3, \dots$ )

$$w = W/2$$

$$W_v = v \pi / W \quad (v = 0, 1, 2, 3, \dots)$$

The Fourier transform of the current distribution is

$$\tilde{J}_y(\xi) = \frac{2\pi}{k_0 d} \sum_{n=-\infty}^{+\infty} \delta(\xi - k_z - \xi_n)$$

$$+ \sum_{v=0}^{+\infty} A_v F_v(\xi_n) \quad (3)$$

where

$$F_v(\xi_n) = \frac{\pi}{2} (J_0(\xi_n w) + \frac{v \pi}{2})$$

$$+ (-1)^v J_0(\xi_n w - \frac{v \pi}{2})$$

$$\xi_n = \frac{2n\pi}{d}$$

$J_0(x)$ : the Bessel function

$\delta(x)$ : the delta function.

The total field is sum of the electric fields generated by the periodic array of strips. Therefore the transform of the total field is

$$\tilde{E}(\xi) = \tilde{G}(0, \xi) \tilde{J}_y(\xi) \quad (4)$$

The total  $E$ -field on any strip is equated to zero. The structure shown in Fig. 1 is a periodic one. By Floquet's theorem, it is enough to consider a single period, for example  $n=0$ . So that

$$\int_{-\infty}^{+\infty} \tilde{E}(z) \Psi(z) dz = 0$$

$$= \int_{-\infty}^{+\infty} \tilde{E}(0, t) \frac{\sin(t - \xi)}{t - \xi} dt = 0 \quad (5)$$

where  $\Psi(z) = \exp(j\xi z) (U(z+w) - U(z-w))$

$U(x)$ : the unit step function.

Substituting (4) into (5) we obtain

$$\sum_{n=-\infty}^{+\infty} \Phi(\xi_n, k_z) \frac{\sin(k_z + \xi_n - \xi) w}{(k_z + \xi_n - \xi) w} = 0 \quad (6)$$

where

$$\Phi(\xi_n, k_z) = \sum_{v=0}^{+\infty} \frac{A_v F_v(\xi_n)}{j\eta + \xi \operatorname{ctg}(\xi h)}$$

$$\eta = \sqrt{k_0^2 - (k_z + \xi_n)^2}$$

$$\xi = \sqrt{\epsilon_r k_0^2 - (k_z + \xi_n)^2}$$

According to sampling theorem Eq. (6) provides us infinite sets algebraic equations

$$\sum_{v=0}^{1-1} B_{v, \mu} A_v = 0 \quad (\mu = 0, 1, 2, \dots) \quad (7)$$

where

$$B_{\mu, v} = \sum_{n=-m}^m \tilde{G}(\xi_n, k_z) F_v(\xi_n) \Omega(\xi_n)$$

$$\Omega(\xi_n) = \frac{\xi_n w \sin((\mu\pi/2) - \xi_n)}{(\mu\pi/2)^2 - (\xi_n w)^2}$$

Eq. (7) is sets of homogeneous equation. To make  $A_r$  having a nontrivial solution, the determinant of Eq. (7) must equate zero

$$|B_{r,v}| = 0 \quad (8)$$

Eq. (8) is the relation we are seeking. Next we express  $k_z$  in terms of the angles  $\theta_m$ ,  $\theta_{1h}$ ,  $\theta_{2h}$ . The structure shown in Fig. 1 is designed such that  $d \geq 0.5\lambda_0$ , and only  $\beta_{-1}$  is in the visible region where  $\lambda_0$  is free-space wavelength. By theory of traveling wave antennas (3), the pattern of the structure in the x-z plane is

$$|E(\theta)| \propto \frac{1}{\sqrt{a^2 + (\beta_{-1} - k_0 \cos \theta)^2}} \quad (9)$$

From the equation (9) we may find the relation between  $\beta$ ,  $a$ , and  $\theta_m$ ,  $\theta_{1h}$ ,  $\theta_{2h}$ . They are

$$\beta = k_0 \cos \theta_m + \frac{2\pi}{d} \quad (10)$$

$$a = -\frac{1}{2} |\cos \theta_{1h} - \cos \theta_{2h}| \quad (11)$$

Substituting (10), (11) and parameters  $W$ ,  $d$ ,  $h$ ,  $k_0$  into (9), we can yield the desired complex dielectric constant of slab.

## RESULTS

Complex dielectric constant of two different slab-coated copper foils have been measured. One of them is FBD-2 slab-coated copper foil. The leaky-wave structure shown in Fig. 1 is made of FBD-2 slab-coated copper foil,  $d=5.6\text{mm}$ ,  $W=1.6\text{mm}$ ,  $h=3\text{mm}$ ,  $L=200\text{mm}$ . The angles  $\theta_m$ ,  $\theta_{1h}$ ,  $\theta_{2h}$  of measured patterns of the structure for different frequencies are listed Table 1. The complex dielectric constant  $\epsilon_r$ ,

computed in terms of  $\theta_m$ ,  $\theta_{1h}$ ,  $\theta_{2h}$  are given in same table. The complex dielectric constant of other slab-coated copper foil is listed in table 2. Here the geometry of the leaky wave structure is that  $d=3\text{mm}$ ,  $W=1\text{mm}$ ,  $h=1.5\text{mm}$ ,  $L=100\text{mm}$ . The angles  $\theta_m$ ,  $\theta_{1h}$ ,  $\theta_{2h}$  of measured patterns of the structure are listed in Table 2, too. Error in  $\epsilon_r$  depend on measuring error in the angles  $\theta_m$ ,  $\theta_{1h}$ ,  $\theta_{2h}$ . Here  $|\Delta \theta_m| = 0.5^\circ$ ,  $\Delta \theta_{1h} = \Delta \theta_{2h} = \pm 0.1^\circ$ , relative errors in  $\text{Re}(\epsilon_r)$  and  $\text{tg}\delta$  are about 2% and 10%, respectively. This method is simple and can measure the complex dielectric constant of a thick dielectric slab-coated conductor in millimeter waves and can realize automatical measurement.

## REFERENCES

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Table.1

$f$ (Gc)	$\theta_m$	$\theta_{1h}$	$\theta_{2h}$	$\text{Re}(\epsilon_r)$	$\text{tg}\delta$
35.28	106.3°	105.1°	107.6°	$2.92 \pm 0.06$	$(5.3 \pm 0.5) \times 10^{-3}$
36.76	99.6°	98.2°	101.3°	$2.90 \pm 0.06$	$(6.0 \pm 0.6) \times 10^{-3}$
37.42	97.0°	95.6°	98.4°	$2.90 \pm 0.06$	$(5.2 \pm 0.5) \times 10^{-3}$

Table.2

$f$ (GC)	$\theta_m$	$\theta_{1h}$	$\theta_{2h}$	$\text{Re}(\epsilon_r)$	$\text{tg}\delta$
35.05	62.2°	59.0°	65.0°	$16.5 \pm 0.3$	$(8.4 \pm 0.8) \times 10^{-3}$
36.04	54.8°	50.1°	57.3°	$16.4 \pm 0.3$	$(11.4 \pm 1.1) \times 10^{-3}$

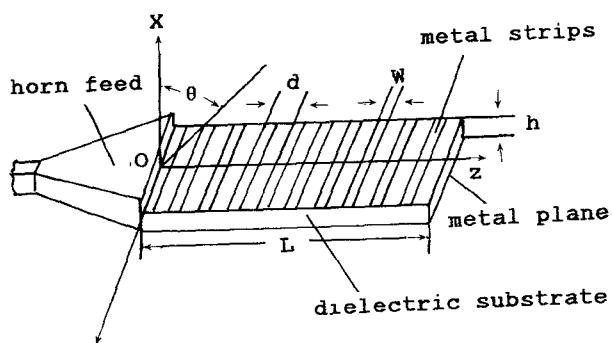


Fig. 1

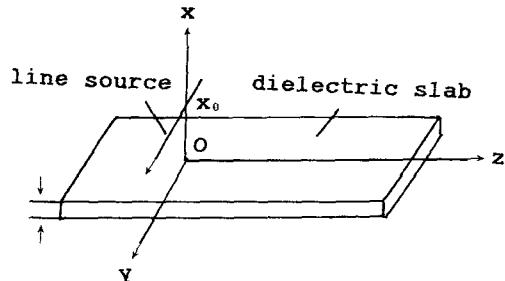


Fig. 2